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FEATURES OF MODELING INVESTMENT PORTFOLIO RISKS UNDER CRISIS CONDITIONS

ОСОБЛИВОСТІ МОДЕЛЮВАННЯ РИЗИКІВ ІНВЕСТИЦІЙНОГО ПОРТФЕЛЯ В УМОВАХ КРИЗОВИХ ЯВИЩ

The main goal of investment activity is to minimize the level of investment risk while finding the optimal balance between return and risk. This process involves accounting for probabilistic factors that arise from the inherent uncertainty in financial activities. Investors are constantly faced with the challenge of managing these risks, particularly in the context of financial markets that exhibit significant volatility and unpredictability. In this regard, the search for models that allow for accurate risk assessment and management is a key aspect of modern portfolio theory. Currently, one of the most widely used models for solving the portfolio optimization problem is the Harry Markowitz model, often referred to as Modern Portfolio Theory (MPT). The Markowitz portfolio optimization model seeks to achieve two possible outcomes: either by minimizing the variance (or risk) of the portfolio's return at a given level of expected return, or by maximizing the expected return at a given level of variance (risk). Despite the obvious advantages of the Markowitz model, such as its widespread applicability and relative ease of implementation in practice, it has several notable limitations. One key drawback is that the model uses variance as a measure of risk, which treats deviations from the expected return symmetrically. This means that both positive (upward) and negative (downward) deviations are considered equally risky, which contradicts the typical investor's view, as they are more concerned with downside risk rather than upside potential. Another limitation of the Markowitz model is the assumption of normally distributed asset returns. In reality, financial markets do not always conform to this assumption. Extreme returns, often referred to as "fat tails," tend to occur more frequently than what would be predicted by a normal distribution. In response to these limitations, there has been growing interest in the development of alternative models that can more accurately capture the complexities of financial markets. One such approach is the use of stochastic models that take into account the time-varying nature of asset volatility and the impact of extreme events. These models seek to minimize portfolio investment risk by incorporating more realistic assumptions about market behavior, including the presence of volatility clustering and non-normal return distributions. The article presented is focused on a stochastic model aimed at minimizing portfolio investment risk. This model addresses some of the shortcomings of the traditional Markowitz framework by better accounting for the unpredictable and often turbulent nature of financial markets. In particular, it has been shown that systematic risk factors play a dominant role in shaping the expected return of an investment portfolio, especially in economies undergoing transition or transformation.

Key words: risk evaluation, portfolio investments, Markowitz model, probit regression, logit model.

Основною метою інвестиційної діяльності є мінімізація рівня інвестиційного ризику та знаходження оптимального балансу між прибутковістю та ризиком. Цей процес передбачає врахування ймовірнісних факторів, що виникають через властиву фінансовій діяльності невизначеність. Інвестори постійно стикаються з викликом управління цими ризиками, особливо в контексті фінансових ринків, які характеризуються значною волатильністю та непередбачуваністю. У зв'язку з цим, пошук моделей, що дозволяють точно оцінювати та управляти ризиками, є ключовим аспектом сучасної теорії оптимізації портфеля. На сьогодні одним із найбільш поширених підходів до розв'язання задачі оптимізації портфеля є модель Гаррі Марковіца, також відома як modern portfolio theory (MPT). Модель Марковіца для оптимізації портфеля має на меті досягнення одного з двох результатів: або мінімізувати дисперсію (ризик) прибутковості портфеля при заданому рівні очікуваної прибутковості, або максимізувати очікувану прибутковість при заданому рівні дисперсії (ризику). Попри очевидні переваги моделі Марковіца, зокрема її широке застосування та відносну простоту впровадження на практиці, вона має кілька суттєвих обмежень. Одним із ключових недоліків є те, що модель використовує дисперсію як міру ризику, яка однаково враховує як позитивні (зростання), так і негативні (спад) відхилення від очікуваної прибутковості. Це суперечить типовому погляду інвестора, який більше стурбований ризиком спадання, а не можливістю зростання. Інше обмеження моделі Марковіца полягає у припущенні нормального розподілу доходностей активів. Однак у реальних фінансових ринках це припущення не завжди підтверджується. Екстремальні доходності – так звані "товсті хвости" – трапляються частіше, ніж це передбачено нормальним розподілом. У відповідь на ці обмеження зростає інтерес до розробки альтернативних моделей, які здатні точніше відображати складність фінансових ринків. Одним із таких підходів є використання стохастичних моделей, які враховують змінність волатильності активів з часом та вплив екстремальних подій. Ці моделі прагнуть мінімізувати інвестиційний ризик портфеля, включаючи більш реалістичні припущення про поведінку ринку, зокрема наявність кластеризації волатильності та ненормальних розподілів доходностей. У представленій статті увагу приділено стохастичній моделі, що спрямована на мінімізацію інвестиційних ризиків портфеля. Ця модель вирішує деякі недоліки традиційної моделі Марковіца, краще враховуючи непередбачувану і часто нестабільну природу фінансових ринків. Зокрема, було показано, що фактор систематичного ризику відіграє домінуючу роль у формуванні очікуваної прибутковості інвестиційного портфеля, особливо в економіках, що трансформуються.

Ключові слова: оцінка ризику, портфельні інвестиції, модель Марковіца, пробіт-регресія, логіт-модель.

Statement of the problem. The acceleration of globalization processes over the past decade has led to the formation of a new global financial architecture, the key determinants of which are a significant acceleration of capital flows, dynamic growth in the volume of capital redistribution through financial markets, and the widespread use of securities trading operations as an investment channel. This allows mobilizing temporarily free financial resources of investors regardless of their location. At the same time, the functioning of financial markets both at the level of individual countries and on a global scale leads to the emergence of a wide range of various risks that need to be promptly identified, their concentration levels predicted, and a system of measures for their possible neutralization developed. Moreover, in the context of increasing crises in financial markets under the influence of global risks, transformation processes are significantly accelerated, signals and behavioral reactions appear that financial markets have not encountered before. In light of the above, the problems of assessing the risks of portfolio investments in response to modern challenges, as well as the formalization of the risk assessment process using economic and mathematical methods maximally adapted to the changing conditions of financial markets, are becoming especially relevant. These methods can form the basis for developing algorithms for preventive measures and behavioral strategies for rapid and timely response.

Analysis of recent research and publications. The existing shortcomings of Markowitz's portfolio theory have spurred the emergence of a large number of modified models, which have significantly increased the practical possibilities of portfolio investing at each historical stage. Modern approaches to assessing financial risks in the formation of investment portfolios in unstable environments involve the use of elements from probability theory and mathematical statistics to formulate an adequate model of stock market processes, estimate their parameters, and develop recommendations for making investment decisions.

Among the most interesting contemporary approaches, it's worth noting the modern stochastic methods proposed by J. Bishwal [3] and D. Dilger [4], who suggested evaluating the risk behavior of investors using mainstream theories of payment obligation assessment. Portfolio investments, as is well known, are described by multifactor models. For instance, some authors [5] have proposed alternative characteristics of the nature of risk and developed a model that differs from the classical models of William Sharpe and Harry Markowitz, allowing the use of stochastic analysis methods for calculating the profitability of risky operations. Promising unifying models at present include those proposed by A. Matviychuk [6], V. Ankhom [7], and S. Berzin [8], which offer innovative methodologies for developing probabilistic models for evaluating the profitability of investment decisions.

The Ukrainian economic school has introduced certain nuances in the interpretation of the concepts of “risk” and “investment portfolio.” Among the works of Ukrainian authors, it is worth highlighting the research [9] that examines various mechanisms for predicting changes in systematic and specific risk upon the occurrence of certain events. This research has laid the foundation for developing a model of investment portfolio risks in the context of a global crisis.

Formulation of the research task. In the context of the dynamic processes occurring in the market, the importance of researching risk factors and, consequently, the returns of the investment portfolio increases. The objective of this work is to construct a mathematical model that takes into account the influence of stochastic factors on the expected return of an asset. In this regard, the relationship between market conditions and the random component of financial asset returns is described using a probit model. A classification of risks in a transforming economy is also presented.

Summary of the main research material. Due to all its obvious advantages, classical portfolio theory has been recognized worldwide as an important conceptual foundation for scientifically justifying the optimal composition of an investment portfolio based on the assessment of the risk/return ratio. At the same time, it is important to consider that the MPT model is based on a number of assumptions that have lost their relevance in modern conditions: all investors simultaneously seek to maximize expected return (or economic utility); all investors have the same investment time horizon; all investors are risk-averse and rational; all investors make investment decisions solely based on expected return and risk; all investors have access to the same information; and taxes, transaction costs, and other factors are not taken into account.

The main shortcomings of the MPT model have been systematically organized by us in relation to the aforementioned assumptions in Table 1.

It is worth noting that scientific interest in researching the volatility of financial instruments and selecting adequate tools for assessing investment risks has significantly increased as global crises deepen. Based on a review of the latest publications by Ukrainian and foreign authors dedicated to the study of mathematical models for assessing the risks of portfolio investments, an important conclusion can be drawn: the classical MPT model can only be applied in markets with absolute capital protection.

However, in today's rapidly changing global environment, investors continuously encounter risks that often exceed the risks associated with portfolio investing. The accumulation of systematic risks complicates the proper application of classical models and requires the development of a specific mathematical apparatus to mitigate the negative factors stemming from the assumptions underlying these models, which are far from reality. Therefore, constructing new and improving existing mathematical models that minimize risks is a pressing task in economic-mathematical modeling. Taking these important findings into account, Sharpe developed the Capital Asset Pricing Model (CAPM), which helps make more informed investment decisions based on the analysis of the relationship between risk and return in an equilibrium market.

In the context of the researched problem, conceptually significant scientific achievements of classical portfolio theory include the recognition that risk associated with investing in financial instruments is just as important a characteristic as expected investment return. The Modern Portfolio Theory (MPT) allows for the assessment of risks relative to the expected level of return and facilitates the most effective decision-making regarding the allocation

Table 1

Main Assumptions of Modern Portfolio Theory and Their Shortcomings

Assumption	Shortcomings of the Assumptions
All investors seek to maximize expected return (or economic utility).	According to this assumption, investors aim to maximize economic utility to achieve the highest expected return on their invested capital (as much as possible) regardless of other considerations. This is a key assumption of the efficient market hypothesis on which modern portfolio theory is based. However, in reality, markets are not always efficient, and the goals of investors can vary significantly.
All investors have the same investment time horizon.	This assumption suggests that all investors have the same timeframe for their investments, influencing their risk tolerance and investment strategy. In reality, investors often have varying investment horizons – some may invest for the short term, while others may have a long-term focus. Moreover, investors can change their initial plans and adjust their investment horizons depending on market conditions. This variability can significantly affect decision-making and the types of assets chosen for a portfolio.
All investors are risk-averse and rational.	This assumption suggests that investors are inclined to take on higher risk only when offered a correspondingly higher expected return. This is a key assumption of the efficient market hypothesis. However, findings from behavioral economics indicate that market participants often behave irrationally and are sometimes compelled to pay a premium for risk. This means that their risk tolerance can be inconsistent and influenced by emotions, market sentiment, and other non-rational factors.
All investors make decisions solely based on expected return and risk.	This implies that the utility curve is a function of expected return and expected variance (or standard deviation) of returns. It assumes that, for a given level of risk, investors prefer more profitable portfolios. Similarly, at a specified expected return, investors favor lower risk. In these assumptions, an individual asset or portfolio is considered efficient if no other asset or portfolio offers a higher expected return for the same (or lower) risk or lower risk for the same (or higher) expected return. However, this assumption often fails in practice, especially during financial crises when all assets tend to have positive correlations, causing them to change (decrease) proportionally.
All investors have access to the same information.	This assumption is also a premise of the efficient market hypothesis, which posits that all investors can access the same information to make decisions. In reality, financial markets are characterized by information asymmetry, insider trading, and participants who are better informed than others. Such disparities can lead to significant advantages for certain investors, skewing the market dynamics and undermining the premise that all investors are on equal footing regarding information. This can result in inefficient pricing of assets and challenges in achieving optimal portfolio decisions.
Calculations are made without considering taxes and transaction costs.	This assumption leads to a significant simplification of modern portfolio theory. In reality, financial assets are subject to taxation and operational costs (such as commissions paid to brokers and dealers), which can substantially affect investment returns. Ignoring these factors may lead investors to select a different combination of assets within their investment portfolios, potentially resulting in less optimal decisions. Thus, a more comprehensive approach that incorporates these considerations is essential for accurately assessing investment performance.

Source: compiled by the author

of individual investment assets within an investment portfolio, both for individual and institutional investors.

Thus, Harry Markowitz's classical model is an effective assessment tool that helps make decisions regarding the selection of investment assets for a portfolio, which collectively have a lower risk than any individual asset. It is important to note that the MPT model shifts the focus from analyzing the characteristics of individual investments to evaluating the statistical dependencies between specific securities that make up the investment portfolio. In other words, according to contemporary portfolio theory, the object of investment management is the entire portfolio, and investments should be managed as a portfolio rather than as individual financial instruments that comprise it.

Ukraine is increasingly integrating into the system of global economic relations, which leads to fluctuations in the returns of investment portfolios. The transformation process is complicated by the presence of stochastic situations with varying assessments of the likelihood of their occurrence. The resulting uncertainty in business conditions defines an increased risk in investment activities. In modern conditions, transformational risks also arise, which can be mathematically characterized as an assessment of the amplitude of fluctuations in the expected return of an investment portfolio in relation to the transformation process, characterized by a high degree of heterogeneity and uncertainty. The main types of risks associated with investment portfolios are presented in Table 2.

It should also be noted that in the context of economic globalization, an investment portfolio is formed in a stock market characterized by an increased level of risks due to the presence of a significant share of transformational risk elements. This is influenced by investors' access to information that is unavailable or only partially accessible to all participants in the process. Therefore, when modeling investment risks, it is essential to construct the model in such a way that it comprehensively assesses situations with maximum uncertainty.

As previously mentioned, there is a stochastic dependence between the forecasted indicators. In constructing the model, we will assume that there is a certain one-time action factor that occurs at a random moment in time. Let there be certain investments in the m-th asset that allow for a certain return S_m We will represent the mathematical model of this random process as follows:

$$T(t) = S_m + \theta_m y_{mt} + \gamma_{mt}, \tag{1}$$

where the following notations are introduced:

S_m – the return of the m -th asset at a certain moment in time, y_{mt} – discrete random variable that takes on values. ±1 ($+1$, if the return is higher than the average value, and vice versa), θ_m – function of change S_m. It should be noted that in the presented formula (1), the quantity γ_{mt} characterizes the impact of random factors.

The function θ_m for each asset m characterizes the average deviation of the actual return from its expected value, and is defined by the formula.

$$\theta_m = \frac{1}{\mu}\sum_m^{\mu}\left(T(t) - S_m\right) = \begin{pmatrix} 1, if\ T(t) - S_m > 0 \\ 0, if\ T(t) - S_m = 0 \\ -1, if\ T(t) - S_m < 0 \end{pmatrix} \tag{2}$$

In this case, based on equation (2), the function θ_m also takes values of ±1, indicating the influence of a certain random factor on the return function

Assuming that market changes affect the probability of changes in the return of a financial asset, we can represent this process with a probit regression model in the following form:

$$y_{mt} = \mathbb{E}\left(\frac{y_{jt}}{\beta_t}\right) + \gamma_{it}, \tag{3}$$

where

$$\mathbb{E}\left(\frac{y_{jt}}{\beta_t}\right) = 1 \cdot F^o\left(\beta_t^* \alpha\right) - 1 \cdot \left(1 - F^o\left(\beta_t^* \alpha\right)\right) = F^o\left(\beta_t^* \alpha\right)$$

A function that characterizes the probability distribution of the values β_t^* and the random vector α in the stock markets. In this case, the random residuals γ_{it} follow a logistic distribution, which is described by the cumulative function of the form:

$$F^*(\gamma_{it}) = \frac{1}{2} + \frac{1}{2}\tanh(\gamma_{it})$$

Considering the above, we have

$$F^o\left(\beta_t^* \alpha\right) = \frac{e^{(\alpha_0 + \alpha_1\beta_1 + \ldots + \alpha_1\beta_1)}}{1 + e^{(\alpha_0 + \alpha_1\beta_1 + \ldots + \alpha_1\beta_1)}} \tag{4}$$

It should be noted that the function y_{mt} in the presented model varies arbitrarily over a certain numerical interval and is nonlinear. This function is not defined analytically, but is constructed using specific computer technologies, making the calculation process quite labor-intensive

In practice, the function $F^o\left(\beta_t^* \alpha\right)$ *is chosen as the standard normal distribution function with parameters* β_t^*, α. It should be noted that the search for an adequacy estimate of this model remains relevant, and the issue of the accuracy of parameter estimation in the model is quite controversial. The most commonly used criterion for this is the McFadden's R^2 criterion, which evaluates the maximum likelihood that characterizes the joint density distribution of the observed random variables.

A significant simplification of the considered model is obtained when the mathematical expectation (mean value) of the return is given by the following expression:

Table 2

Classification of Investment Portfolio Risks

Classification of Investment Portfolio Risks			
Systematic Risks		**Specific Risks**	
Classical Systematic Risk - Risk that cannot be eliminated through diversification; - Risk that is caused by the overall movement of the market;	Systematic Transformation Risk - Market risk generated by macroeconomic events; - Risk associated with the economic situation;	Classic Specific Risk - Diversified Risks;	Specific Transformational Risk - Risk resulting from maximum uncertainty; - Risk that can be eliminated through portfolio diversification;

Source: compiled by the author

$$\mathbb{E}\left(S_m+\theta_m y_m+\gamma_m\right)=S_m+\theta_m\left(2F^o\left(\beta_t^*\alpha\right)-1\right) \quad (5)$$

Considering (5), we see that the expected growth of the return function S_m is possible when $2F^o\left(\beta_t^*\alpha\right)-1=0$, or when

$$F^o\left(\beta_t^*\alpha\right)<\frac{1}{2}.$$

Interpreting variance as a measure of risk, we obtain the following expression.

$$D=\theta_m^2\,(4F^o\left(\beta_t^*\alpha\right)\left(1-F^o\left(\beta_t^*\alpha\right)\right)+D_{\gamma_m}, \quad (6)$$

From which it follows that the value D_{max} is obtained when $F^o\left(\beta_t^*\alpha\right)=\frac{1}{2}$.

Considering the presented model, for example, in the case of a portfolio with three financial assets, we have the expression for determining the mathematical expectation of returns:

$$\mathbb{E}\left(S_m\right)=\sum_{m=1}^{3}\beta_m\alpha_m+\beta_1\theta_1\,(3F^o\left(\beta_t^*\alpha_1-1\right)+$$
$$+\beta_3\alpha_3\left(3F^o\left(\beta_3\alpha_3\right)-1\right)$$

Thus, we see that the return on the investment portfolio is entirely determined by the parameters $\beta_m, m=1,2,3$ and α_m These parameters characterize the returns, the first of which is formed based on the mathematical expectations of the asset returns, while the second represents the risk and should be minimized by the investor as much as possible.

Conclusions. The model obtained in the study allows, unlike existing ones, to present the risk of an investment portfolio in the form of two components. The estimation of the variance of asset returns has been found. It is shown that the factor of systematic risk generally dominates the process of forming the expected return of the investment portfolio in a transforming economy. The presence of a high level of risk is characterized by the value of the parameter β_m.

Considering the obtained expression (6), the risk in the constructed probit model is quite predictable. The presented model allows the investor to independently identify risk factors and forecast changes in portfolio risks in the future. This model is sufficiently universal for any securities market with open information about the history of price movements and can be adjusted according to the utility function of each investor.

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